Feeding black holes at galactic centres by capture from isothermal cusps

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Abstract

The capture rates of stars and dark particles onto supermassive black holes depend strongly on the spatial and kinematical distribution of the stellar and dark matter at the centre of bulges and elliptical galaxies. We here explore the possibility that all ellipticals/bulges have initially isothermal cusps ($\rho \propto r^{-2}$). If the orbits can be adequately randomized a significant fraction of the total mass of black holes in the bulges of galaxies will be due to the capture of stars and dark matter. The dark matter fraction of the total mass captured may be as high as 20–40 percent for typical cold dark matter halos. A tight relation $M_\bullet \sim 10^8(\sigma_*/200\,\mathrm{km\,s^{-1}})^5$ between black hole mass and stellar velocity dispersion can arise at the high mass end ($M_\bullet \geq 10^8 M_\odot$) if these giant black holes grow primarily by the capture of stars without tidal disruption. For smaller black holes a shallower $M_\bullet - \sigma_*$ relation with larger scatter is expected. Efficient randomization of the orbits can be due to remnant accretion discs or the dense central regions of infalling satellites which can avoid tidal disruption and sink to the sphere of influence by dynamical friction. The presence of an isothermal cusp and the reduction of the relaxation time scale at the sphere of influence enhance the estimated tidal disruption rate of stars to $\sim 10^{-4} - 10^{-2}\,\mathrm{yr}^{-1}$ per galaxy. Disruption flares in bright galaxies may thus be as frequent as a few percent of the supernovae rate at moderate redshifts when the galaxies still had an isothermal cusp. The efficient replenishment of the loss cone also explains why the supermassive binary black holes expected in hierarchically merging galaxies do generally coalesce as suggested by the observed relation between black hole mass and the inferred mass of stars ejected from an isothermal cusp.

Key words: Black hole physics - quasars: general - galaxies: kinematics and dynamics - galaxies: interactions - galaxies: halo - dark matter
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1 Introduction

The confidence in the measurement of the masses of supermassive black holes in nearby galaxies has significantly increased in the last couple of years (see Merritt & Ferrarese 2001 for a recent review). This is mainly due to the newly established correlation between black hole mass and velocity dispersion of the bulge of the host galaxy (Gebhardt et al. 2000, Ferrarese & Merritt 2000). Most if not all galactic bulges appear to contain a black hole with mass $M_\bullet \propto \sigma_*^{4-5}$. For the published samples of reliable black hole masses this correlation appears much tighter than that between bulge mass and bulge luminosity (Kormendy & Richstone 1995, Magorrian et al. 1998). A number of suggestions have been made which can explain the slope of the correlation (Silk & Rees 1998; Haehnelt, Natarajan & Rees 1998; Kauffmann & Haehnelt 2000; Haehnelt& Kauffmann 2000a; Ostriker 2000; Burkert & Silk 2001; Adams, Graf & Richstone 2001) but little has been offered to explain its apparent tightness. The physical processes invoked to regulate the black hole mass depend generally on the conditions close to the black hole at radii much smaller than those at which the velocity dispersion of stars is measured. Moreover, the scatter in correlations of observed properties of galaxies like the Tully-Fisher and Faber-Jackson relation is much larger. Haehnelt& Kauffmann (2000b) demonstrated that their model is consistent with the observed scatter. However, the small scatter does not occur naturally in such a model where galaxies build up by hierarchical merging. Should the tightness of the correlation between black hole mass and bulge velocity dispersion stand the test of time a physical mechanism which links the black hole mass and the velocity dispersion of the stars in the bulge more directly may be required. Capture of stars by the black hole for instance is a process that depends straightforwardly on the stellar velocity dispersion (Rees 1988). The main problem is that the orbits of stars with sufficiently low angular momentum are generally assumed to be rapidly depleted, inhibiting efficient growth by accretion of stars (Sigurdsson & Rees 1997, Magorrian & Tremaine 1999, Syer & Ulmer 1999). The density profiles of bright ellipticals exhibit pronounced breaks within which the density profile becomes significantly shallower than isothermal at radii of a few hundred parsecs (Gebhardt 1996). Very little feeding of the black hole can come from stars at large radii, where the loss cone becomes prohibitively small. These shallower cores may however have formed very recently due to the ejection of stars by the supermassive binary black holes (Milosavljević & Merritt 2001, Ravindranath, Ho & Filipenko 2002) expected in hierarchical merging galaxies (Kauffmann & Haehnelt 2000). Strong supports for this idea come from the observed correlation between the mass of the black hole and the mass inferred to be ejected if the galaxy started out with a cusp with a density profile close to isothermal. This makes it likely that bulges form with a stellar density distribution which is close to isothermal all the way down to the sphere of influence of the supermassive black hole. The number of stars

on low-angular momentum orbits and thus the rate of capture of stars by the supermassive at the centre of nearby galaxies must have been much larger prior to the destruction of the isothermal cusp. Here we explore this idea in more detail.

2 Growing supermassive black holes by the capture of stars

2.1 Direct capture and tidal disruption

For supermassive black holes more massive than

$$M_{\rm disr} \approx 10^8 M_\odot \tag{1}$$

direct capture of solar-type stars is possible if the angular momentum of the star is smaller than some critical value (Frank & Rees 1976). For a Schwarzschild black hole this value is given by

$$J_{\rm cap} \leq \frac{lGM_\bullet}{c}, \qquad l = 4; \tag{2}$$

for a Kerr BH, l is slightly > 4 for an incoming particle retrograde to the spin of the hole, and slightly < 4 for a direct particle. In the case of less massive black holes solar-type stars with angular momentum smaller than $\sqrt{2GM_\bullet r_{\rm disr}}$ will be tidally disrupted before they reach the horizon at a radius

$$r_{\rm disr} = \left(\frac{M_\bullet}{M_{\rm disr}}\right)^{-2/3} r_{\rm S}, \tag{3}$$

where $r_{\rm S} = 2GM_\bullet/c^2$ denotes the Schwarzschild radius. The "loss cone" of stars with sufficiently small angular momentum is larger than that for direct capture by a factor $(M_\bullet/M_{\rm disr})^{-2/3}$. Which fraction of the debris can be accreted by the black hole is uncertain and depends on how much gas is expelled by a possible radiation-driven wind (Rees 1988). Dark matter particles with angular momentum as low as given in equation (2) will also be accreted by direct capture. We will discuss this in more detail in section 3.4.

2.2 A simple argument for the $M_\bullet$–σ_* relation

Let us first consider the case of supermassive black holes more massive than $M_{\rm disr}$ embedded in an isothermal stellar cusp ($\rho \propto r^{-2}$). For an isotropic

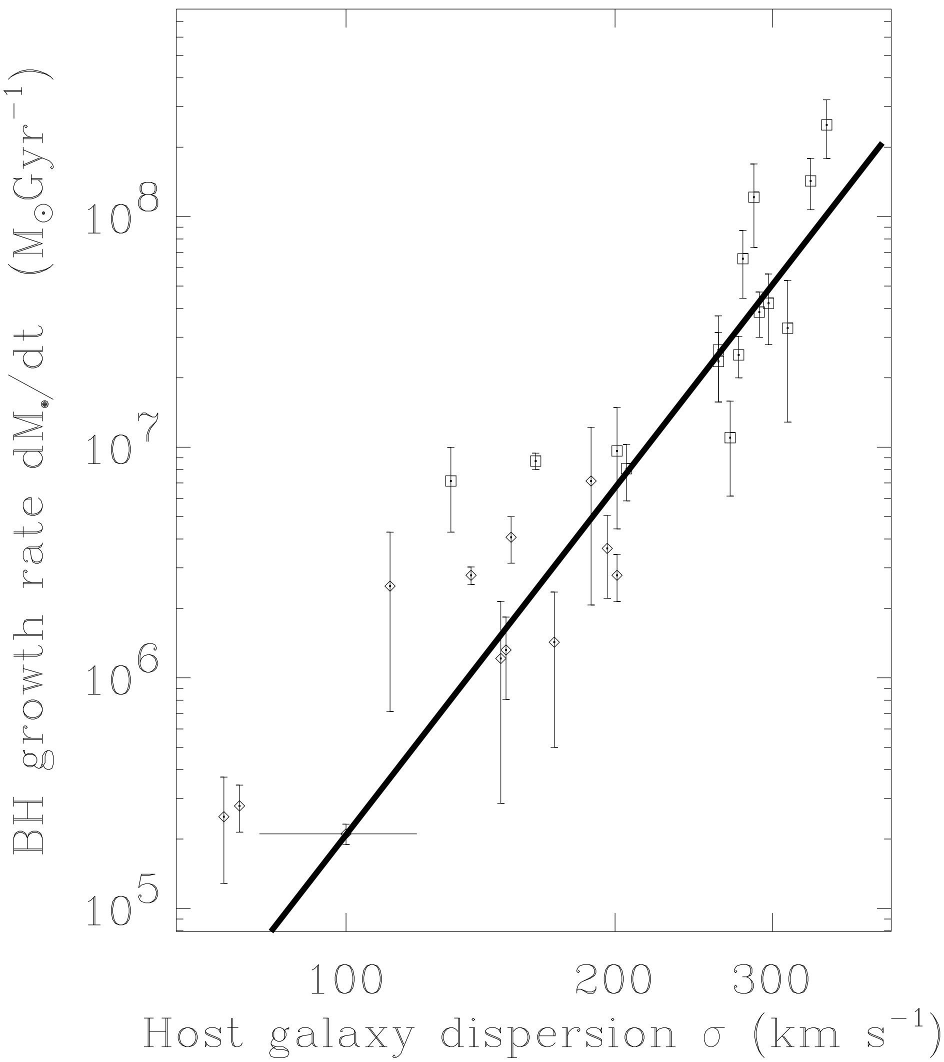

Fig. 1. The rate $\dot{M}_{\rm cap}(r_\bullet)$ for the capture of stars by the black hole as given by equation (8) compared to the growth rate of an observed sample of black holes (Ferrarese & Merritt 2000) time-averaged over a Hubble time of 14 Gyr.

Gaussian velocity distribution a fraction

$$f_{\rm cap}(r) = \left(\frac{J_{\rm cap}}{2\sigma_* r}\right)^2 = \frac{2r_{\rm S} r_\bullet}{r^2} = 4\,\frac{\sigma_*^2}{c^2}\left(\frac{r_\bullet}{r}\right)^2 \tag{4}$$

of the stars at radius r has sufficiently low angular momentum and will be captured within a dynamical time. Here we have defined the radius of the

sphere of influence

$$r_\bullet = \frac{GM_\bullet}{\sigma_*^2} \approx 15 \left(\frac{\sigma_*}{200\,\mathrm{km\,s^{-1}}}\right)^{2.8} \mathrm{pc}. \tag{5}$$

For the right hand side of equation (4) we have assumed the observed relation

$$M_\bullet = 1.4 \times 10^8 M_\odot\,(\sigma_*/200\,\mathrm{km\,s^{-1}})^{4.8} \tag{6}$$

as quoted by Ferrarese & Merritt (2000).

For a (singular) isothermal cusp we can now write the rate at which the "loss cone" of stars with low angular momentum is cleared as a mass accretion rate. of stars captured from radius r. Outside $r_\bullet$ this is given by

$$\dot{M}_{\mathrm{cap}}(r) = 4\pi\,\rho(r)\,r^2 \sigma_*\,f_{\mathrm{cap}} \tag{7}$$

$$= 8\,\frac{\sigma_*^3}{G}\,\frac{\sigma_*^2}{c^2}\left(\frac{r_\bullet}{r}\right)^2. \tag{8}$$

There are a few things to note. First the dynamical time at the sphere of influence is only $t_{\mathrm{dyn}} = r/\sigma_* = 7 \times 10^4 (\sigma_*/200\,\mathrm{km\,s^{-1}})^{1.8}$ yr. To achieve significant growth the "loss cone" has to be refilled many ($\sim f_{\mathrm{cap}}^{-1}$) times. We will come back to this point later. Second the accretion rate at $r_\bullet$ does not depend on mass and decreases as r^{-2} at larger radii. A black hole growing by capture of stars from an isothermal cusp will grow inside-out from the sphere of influence. Due to the strong dependence on radius the mass accretion rate is very sensitive to the stellar distribution just outside the sphere of influence of the black hole. In Figure 1 we compare the accretion rate given by equation (8) with those which would be necessary to grow the black holes in observed galaxies over a Hubble time.

The growth rate will be dominated by capture of stars from $r \sim r_\bullet$ and within a time t_0 the black hole grows to a mass

$$M_\bullet \approx 10^8 M_\odot \left(\frac{\sigma_*}{200\,\mathrm{km\,s^{-1}}}\right)^5 \left(\frac{t_0}{14\,\mathrm{Gyr}}\right). \tag{9}$$

if the loss cone stays full. Slope *and* normalization of the observed $M_\bullet$–σ_* relation can thus be explained by capture of stars if the isothermal cusp existed and the loss cone can be kept filled for a fair fraction of the Hubble time. The velocity dispersion is independent of radius in isothermal stellar systems. The scatter in the observed correlation would then be reduced to that in t_0.

Dynamical relaxation makes stars wander in angular momentum space, and fill the loss cone on a timescale $t_{\rm fill} \sim f_{\rm cap} t_{\rm relax}$ (e.g. Magorrian &Tremaine 1999, Syer & Ulmer 1999). If $t_{\rm fill} > t_{\rm dyn}$ the mass accretion would be controlled by the diffusion rate of stars into the loss cone. The mass accretion rate would be smaller by a factor $t_{\rm dyn}/t_{\rm fill}$ and the mass would be smaller by a factor $t_0/t_{\rm relax}$.

2.3 Why loss cone depletion may not be important

Previous estimates of capture rates of solar-type stars by supermassive black holes are of the order of $10^{-6} - 10^{-4}\,{\rm yr}^{-1}$ (Magorrian & Tremaine 1999, Syer & Ulmer 1999) much lower than required for significant growth of the supermassive black holes in nearby galaxies. These rather low capture rates result mainly from the following assumptions:

- the supermassive black holes are fed from shallow cores typical for bright present-day elliptical galaxies;
- the distribution of the stars feeding the supermassive back hole is spherical or axisymmetric and angular momentum is conserved;
- star-star two body-relaxation is the dominant process for repopulating the loss cone.

However, hierarchical structure formation scenarios suggest that present-day galaxies have evolved strongly with redshift. The first two assumptions are thus most likely not justified for the major fraction of the past history of present-day galaxies. We have already discussed that the shallow cores in bright galaxies may have been established very recently due to ejection of stars by supermassive binary black holes and that all ellipticals/bulges may initially have had a cusp with an isothermal density distribution down to the sphere of influence of the black hole.

The chaotic motions induced by the presence of a black hole have been demonstrated to destroy the triaxiality of galaxies outside the sphere of influence out to about a hundred times $r_\bullet$ in a few crossing times (Gerhard & Binney 1985, Merritt & Quinlan 1998, Sellwood 2001). Poon & Merritt (2002), however, have argued that at smaller radii close to the sphere of influence stable triaxial nuclei may persist. This brings into question to what extent angular momentum in an isothermal cusp around a supermassive black hole would be conserved and whether refilling of the loss cone may occur faster than on a star-star relaxation timescale (*cf* Holley-Bockelmann et al 2002).

For the observed $M_\bullet$–σ_* relation the timescale for the relaxation of the energy of stellar orbits by star-star interactions at the sphere of influence can be

written as

$$t_{\rm relax}(r_\bullet) = \frac{N_*}{8\ln N_*}\, t_{\rm dyn}(r_\bullet) \approx 10 \left(\frac{\sigma_*}{150\,{\rm km\,s^{-1}}}\right)^{6.6} {\rm Gyr}, \tag{10}$$

where N_* and $\ln N_*$ denote the number of stars and the relevant "Coulomb logarithm", respectively (Binney & Tremaine 1987). This relaxation time is longer than a Hubble time for galaxies with $\sigma_* > 150\,{\rm km\,s^{-1}}$, so the amount of black hole growth would be reduced by a factor $(\sigma_*/150\,{\rm km\,s^{-1}})^{6.6}$.

The two-body relaxation time scale is also strongly dependent on the mass of the perturbing objects. The presence of a single heavy perturber of mass $m_{\rm pert}$ in a system of N_* stars will reduce the star-star energy relaxation time scale by a factor $\sim m_{\rm pert}^2/N_* m_*^2$ (see also Polnarev & Rees 1994). At the sphere of influence $N_* m_* \sim M_\bullet$ and the two-body energy relaxation time-scale for an isothermal cusp is reduced below 10 Gyr for mass ratios

$$\frac{m_{\rm pert}}{m_\bullet} \geq 0.0002 \left(\frac{\sigma_*}{200\,{\rm km\,s^{-1}}}\right)^{0.9}. \tag{11}$$

A population of smaller mass perturbers with mass fraction $f_{\rm pert}$ will reduce the relaxation time-scale by a factor $f_{\rm pert}\,(m_{\rm pert}/M_\odot)$ and the two-body energy relaxation time-scale at the sphere influence is reduced below 10 Gyr for a mass fraction

$$f_{\rm pert} \geq 0.007 \left(\frac{m_{\rm pert}}{1000 M_\odot}\right)^{-1} \left(\frac{\sigma_*}{200\,{\rm km\,s^{-1}}}\right)^{6.6}. \tag{12}$$

Note, however, the important process for our models is the relaxation of the angular momentum not energy since stars are captured from the sphere of influence directly. The relaxation of angular momentum is significantly easier to achieve than that of energy and the above timescales should be substantially shorter in the presence of a resonant perturber or a small number of resonant perturbers (Rauch & Tremaine 1996).

2.4 Resonant angular momentum relaxation due to remnant accretion discs

Typical supermassive black holes have accreted a substantial fraction of their mass via an accretion disc The phases of rapid accretion are probably short but when the mass supply from larger radii drains mass accretion rates will drop and the viscous time scales will increase (e.g. Haehnelt & Natarajan & Rees 1998). Phases of rapid accretion should thus leave remnant accretion discs behind. The disc should typically contain molecular gas at a temperature

of 100K. For an α-disc the scale height at the sphere of influence would be about 3×10^{-3} the radius and the viscous accretion time scales would be $\alpha^{-1}(H/R)^{-2} \sim \alpha^{-1}10^5$ times the dynamical time. For a $10^8 M_\odot$ black hole this is longer than the Hubble time for $\alpha \sim 0.3$. For small accretion rates α should be much smaller and for more massive black holes the numbers become even larger. It is thus safe to assume that in phases of low accretion from larger radii remnant accretion discs should survive for a Hubble time. Plausible disc to hole mass ratios would be in the range 10^{-3} to 10^{-2}. Note that a massive disc will be little affected by the drag due to dynamical friction.

The gas in the remnant disc will move on Keplerian orbits in the spherical potential of the black hole. It will act as a massive resonant perturber akin to the collective resonant relaxation due to stars on resonant orbits described by Rauch and Tremaine (1996) and will reduce the time scale for angular momentum relaxation of stars at the sphere of influence to well below a Hubble time. Note again that we do not need to achieve energy relaxation as we have assumed direct capture of stars from the sphere of influence. This is different from the case discussed by Rauch and Tremaine who remarked that loss cone refilling will not be enhanced if energy relaxation is required for stars to migrate to smaller radii. Unlike the time scale for angular momentum relaxation the time scale for energy relaxation is not affected by resonant orbits.

2.5 Perturbing galactic nuclei

The centres of galaxies building up by hierarchical merging are continuously perturbed by a sequence of infalling smaller galaxies. E.g. in the model of Kauffmann et al. (2002) bright galaxies have typically undergone a few mergers with galaxies of 10% their mass and about 10-20 mergers with galaxies 1% their mass since $z \sim 1$. The outer parts of the infalling galaxy will be progressively tidally stripped while it sinks to the centre on a dynamical friction time scale. The mass of the remnant surviving tidal stripping will be $m_{\rm pert} \sim (\sigma_{\rm sat}/\sigma_*)^3\,(r/r_\bullet)\,m_\bullet$ where we have assumed that the density profile of the satellite is also isothermal. The resulting relaxation time scales as $(\sigma_{\rm sat}/\sigma_*)^{-6}$ and at radius r satellites with stellar velocity dispersion

$$\sigma_{\rm sat} \geq 0.06 \left(\frac{r}{r_\bullet}\right)^{\frac{1}{6}} \left(\frac{\sigma_*}{200\,\mathrm{km\,s^{-1}}}\right)^{0.3} \sigma_* \tag{13}$$

reduce the stellar relaxation timescale below 10 Gyr. A significant fraction of the total mass of stars in the cusp may actually be made up from the debris of such tidally disrupted galaxies which then can keep the low-angular momentum orbits continuously populated. The streamers left over from the

tidal disruption of the outer parts of the infalling satellite galaxies will also contribute to the reduction of the relaxation timescale (Tremaine & Ostriker 1999). The massive black holes with masses intermediate between those of stellar and supermassive black holes which may be left over from a pre-galactic episode of star formation would be a plausible population of more numerous less massive perturbers (Madau & Rees 2001). The dynamical friction time scale would be short enough for $1000 M_{\odot}$ black holes to sink to the sphere of influence from about 10 $r_{\bullet}$. A fraction of 10^{-3} of the total mass in the bulge in such intermediate mass black holes would thus also be sufficient to reduce the energy relaxation timescale below a Hubble time. A smaller fraction would be sufficient to reduce the angular momentum relaxation time scale. Note that these black holes are more easily captured into close bound orbits by the central supermassive black holes than stars (Sigurdsson & Rees 1997). Intermediate mass black holes in close orbits around supermassive black holes with $M_{\bullet} \leq 3 \times 10^{6} M_{\odot}$ are good candidates for a detection by LISA (Madau& Rees 2001).

3 Further implications and potential problems

3.1 Other accretion modes

Capture of stars is not the only growth mechanism of black holes. Haehnelt, Natarajan & Rees (1998) have argued (see also Haehnelt & Kauffmann 2001) that the black hole mass density inferred by optical bright accretion is of the same order but somewhat lower than the total mass density inferred from nearby galaxies. This leaves some room for other modes of accretion. There are, however, large uncertainties in both the estimates of the black hole mass density necessary to produce the QSO emission and the total mass density of remnant black holes. Merritt and Ferrarese (2001) e.g. conclude that optical bright accretion in QSOs could account for the total observed black hole mass density in nearby galaxies. Probably at least 30 percent of the total mass density of supermassive black holes in nearby galaxies is already accounted for by accretion of gas (mostly at $z \geq 1.5$). Accretion of gas where the optical emission is obscured by dust will further increase this number (Fabian et al. 1998, Haehnelt et al. 1998). Rather than establishing the $M_{\bullet}$–σ_{*} relation the capture of stars may thus just tighten and refine the exact slope of a pre-existing rough correlation between black hole mass and bulge properties.

3.2 Tidal disruption of stars

So far we have neglected that black holes smaller than $m_{\rm disr} \sim 10^8 M_\odot$ can tidally disrupt stars before they reach the horizon (Rees 1988). This complicates matters as it increases the loss cone by about a factor $(M_\bullet/M_{\rm disr})^{-2/3}$. The increased size of the loss cone should lead to a flattening of the $M_\bullet$–σ_* relation at small masses but probably not as much as to $M \propto \sigma_*^3$. It will be increasingly difficult to keep the enhanced loss cone continuously filled at smaller masses for a Hubble time. Furthermore, hierarchical galaxy formation predicts the merger rates to decrease rapidly with decreasing bulge luminosity (Kauffmann et al. 2002). The scatter of the $M_\bullet$–σ_* relation is therefore expected to increase at smaller masses. The number of black hole mass determinations in fainter galaxies is small and observational constraints are still weak. We further note that if loss cone refilling is as efficient as assumed here then the rates for flares due to the tidal disruption of stars in bright galaxies will be significantly larger at moderate redshift than nearby (Syer & Ulmer 1999, Magorrian & Tremaine 1999). If the mass of tidally disrupted stars is a fraction $f_{\rm disr}$ of the black hole mass then the corresponding rate of flares should be on average a fraction $\sim 0.2 f_{\rm disr}$ of the supernova rate if we assume one supernova per $100 M_\odot$ in stars and $M_\bullet/M_* \sim 0.002$ The peak-luminosities of the flares can reach the Eddington luminosity of the black hole and are likely to significantly exceed those of supernovae. Searches for high-redshift supernovae may thus detect these flares in appreciable numbers.

3.3 Supermassive binary black holes in hierarchically merging galaxies

The black hole hosted by the infalling satellites will form a supermassive binary with the primary black hole. When the circular velocity of the binary equals $\sim \sigma_*$ the binary will harden due to scattering of stars similar to the hardening of binary stars in star clusters. If the binary separation can shrink to a separation

$$
\begin{aligned}
a_{\rm gr} &\sim 4 \times 10^3 \left(\frac{m_1 + m_2}{10^8 M_\odot}\right)^{-1/4} \left(\frac{m_2}{m_1}\right)^{1/4} r_{\rm S} \\
&\sim 4 \times 10^{-2} \left(\frac{\sigma_*}{200\,{\rm km\,s^{-1}}}\right)^{3.6} \left(\frac{m_2}{m_1}\right)^{1/4} {\rm pc}
\end{aligned}
\tag{14}
$$

the supermassive black holes will spiral together within 10 Gyr due to the emission of gravitational waves (Peters 1964). At a separation $a_{\rm gr}$ the loss cone for binary hardening is larger by a factor $\sim 4 \times 10^3 (\sigma_*/200\,{\rm km\,s^{-1}})^{-1.2} (m_1/m_2)^{-1/4}$ than that for direct capture of stars. The evolution time scale is, however, again

set by the stellar relaxation time scale on which low-angular momentum orbits are repopulated. Whether supermassive binary black holes actually reach this separation in normal galaxies has been a matter of debate. Begelman, Blandford, & Rees (1980) have argued that in typical bright elliptical galaxies the timescale is longer than a Hubble time and that accretion of gas would be required to reduce the separation sufficiently for gravitational radiation to become important if loss cone depletion occurs. Yu (2002) in a detailed analysis of a sample of nearby galaxies has shown that the supply of low-angular momentum stars is sufficient to reach $a_{\rm gr}$ if the galaxies are significantly flattened or triaxial. As discussed above, for an isothermal cusp the problem is strongly alleviated (see also Milosavljević & Merritt 2001 and Gould & Rix 2000) and the presence of the dense remnants of infalling satellite galaxies can reduce the stellar relaxation timescale at the sphere of influence, and thus also the coalescence timescale of supermassive binary black holes in bright galaxies, well below a Hubble time. This fits in well with the observed correlation of inferred ejected core mass and black hole mass (Milosavljević & Merritt 2001, Ravindranath et al. 2001).

3.4 Accretion of dark matter

There is good evidence that in the outer parts of bright ellipticals/bulges the gravitational force is dominated by dark matter halos (see Gerhard et al. 2001 for a recent study). However, the mass fraction of dark matter particles in the core of these galaxies is somewhat uncertain and will depend on the density profiles of the dark matter halo and the baryons in the galaxy. The density profiles of stellar nuclei, bulges and dark matter halos can be well described by a double power-law of the form $\rho \propto \tilde{r}^{-\gamma}(1+\tilde{r}^{\alpha})^{\frac{\gamma-\beta}{\alpha}}$, where $\tilde{r}$ is a rescaled radius, while γ, β and α prescribe the steepness of the inner cusp, the outer slope and the sharpness of the transition, respectively (Zhao 1996). For CDM-type structure formation models the dark matter has a cusp $\rho_{\rm DM} \propto r^{-\gamma}$ with γ in the range 1-1.5 (Navarro, Frenk & White 1996; Moore et al. 1998, Klypin et al. 2001). These will, however, be significantly modified when the baryons concentrate at the centre and dominate the gravitational force. This effect can be modelled by adiabatic contraction. If the final potential is isothermal outside $r_\bullet$ due to the formation of the stellar nucleus and bulge the dark matter will attain a power-law distribution with slope $\gamma' = (6-\gamma)/(4-\gamma) = (1.8, 1.66)$, for $\gamma = (1.5, 1)$, respectively. The dark matter fraction of the total mass inside $r_\bullet$ available for capture is given by

$$f_{\rm DM}(r_\bullet) = \frac{M_{\rm DM}(<r_\bullet)}{M_{\rm DM}(<r_\bullet) + M_*(<r_\bullet)} \sim \left(\frac{r_\bullet}{r_{\rm G}}\right)^{2-\gamma'}, \qquad (15)$$

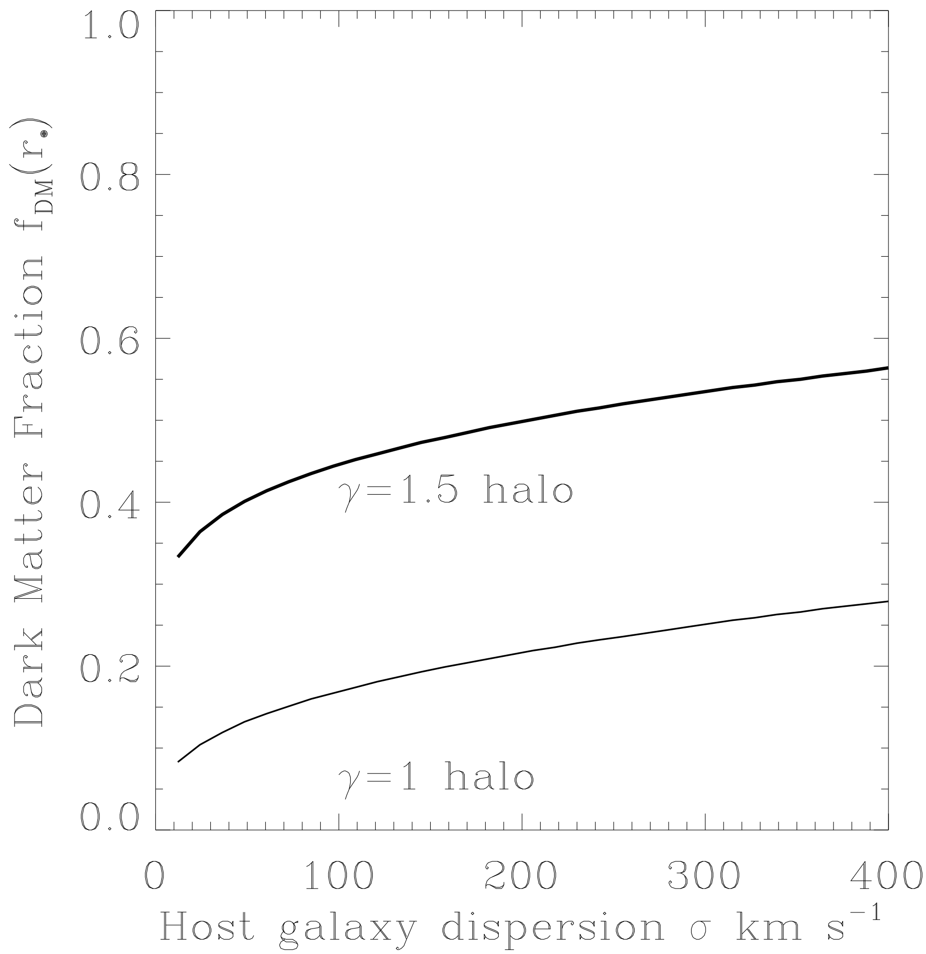

Fig. 2. Dark Matter fraction near the sphere of influence of the black hole. The lower thin line assumes an initial dark matter halo density profile as given by Navarro, Frenk & White (1996) with a $\rho_{\rm DM} \sim r^{-1}$ cusp, and the upper thick line assumes a density profile as given by Moore et al. (1998) with a $\rho \sim r^{-1.5}$ cusp. The halo concentration parameter and the galaxy dispersion are assumed to scale with the halo mass as given in Klypin et al. (2001).

where $r_{\rm G}$ is the radius outside which the dark matter dominates the stars. The fraction $f_{\rm DM}$ is of order 20 percent for $\gamma = 1.5$ and is rather insensitive to $r_{\rm G}$ for a reasonable range of 5-50 kpc.

Figure 2 shows the dark matter fraction at $r_\bullet$ for an isothermal stellar distribution adiabatically grown within more realistic dark matter halo profiles (see Ullio, Zhao & Kamionkowski (2001) for details of the calculation). The values of the dark matter fraction are in the range of 20-50 percent. The upper end is probably already too large to be consistent with the mass-to-light ratios in elliptical galaxies (Gerhard et al. 2001). Note that the adiabatic contraction

probably overestimates the dark matter fraction because of the hierarchical build-up of the galaxies by merging (Ullio, Zhao & Kamionkowski 2001).

For black holes with masses $M_\bullet > M_{\rm disr}$ the dark matter particles will be captured in the same way as the stars. A fraction $f_{\rm DM}$ of the total mass captured will be dark matter particles. It is interesting to note that for a given density distribution the capture rate of collisionless dark matter particles is the same as that of collisional dark matter particles if the loss cone can be kept continuously filled. Of course if the dark matter cross section is large enough that the dark matter becomes fluid-like then the inner dark matter density distribution may be modified (Ostriker 2000). The dark matter particles will also contribute a fraction $f_{\rm DM}$ to the hardening of supermassive binary black holes and will be ejected from the core in the same manner as stars.

4 Conclusions

The capture of stars and dark matter particles from orbits with sufficiently low angular momentum to pass the event horizon contributes significantly to the total mass of black holes in the bulges of galaxies if all bulges initially had an isothermal cusp and if no depletion of these orbits occurs. The dark matter fraction of the total mass captured is 20–40 percent for typical CDM-like halos. A tight relation between black hole mass and stellar velocity dispersion of the form $M \propto \sigma_*^5$, very similar to the observed relation, arises if the black holes in the bulges of galaxies gain most of their mass by this mechanism. The relation is then expected to flatten at black holes masses smaller than $10^8 M_\odot$ where capture and tidal disruption of stars outside the horizon becomes important. Accretion of gas during the active QSO phase with its peak at redshift $z \sim 2-3$ already accounts for a significant fraction ($\geq 30\%$) of the total mass density in black holes. The capture of stars may thus tighten and refine the exact slope of a pre-existing rough correlation between black hole mass and bulge properties. It will thus be interesting to see if the tightness of the correlation persists when sample sizes get larger.

In our model bright ellipticals initially had isothermal cusps and stars are predominantly captured from the sphere of influence. Relaxation of energy momentum is not required to fill the loss cone in these isothermal cusps. The randomization of stellar orbits is enhanced over the classical two-body relaxation because of processes such as the presence of long-lived remnant accretion discs left behind from phases of rapid accretion, the infall of smaller galaxies predicted by hierarchical models of galaxy formation and perhaps also the sinking of massive black holes left over from a pre-galactic episode of star formation.

The presence of an isothermal cusp and the repopulation of low-angular momentum orbits will considerably increase the disruption rate of stars by supermassive black holes in the bulges of bright galaxies at moderate redshifts. Searches for high-reshift supernovae may thus detect disruption flares as frequently as a few percent of the supernovae rate. The presence of an isothermal cusp and efficient loss cone refilling will also accelerate the hardening of supermassive binary black holes so that they will generally reach the separation where they can spiral together within a Hubble time due to the emission of gravitational waves. The latter strongly supports the idea that the cores of elliptical galaxies have formed very recently due to ejection of stars by supermassive binary black holes.

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References

[1] Adams F.C., Graff D.S., Richstone D.O., 2001, ApJ, 551, L31

[2] Begelman M., Blandford R., Rees M.J., 1980, Nature, 287, 307

[3] Binney J., Tremaine S., 1987, Galactic Dynamics, Princeton University Press, Princeton

[4] Burkert A., Silk J., 2001, ApJ, 554, L151

[5] Fabian A.C., Barcons X., Almaini O., , Iwasawa K., 1998, MNRAS, 279, L11

[6] Frank J. , Rees M.J., 1976, MNRAS, 176, 633

[7] Ferrarese L., Merritt D., 2000, ApJ, 539, L9

[8] Gebhardt K., et al. 1996, AJ, 112, 105

[9] Gebhardt K. et al. 2000, ApJ, 539, L13

[10] Gerhard O., Binney J., 1985, MNRAS, 216, 467

[11] Gerhard O., Kronawitter A., Saglia R.P., Bender R., 2001, AJ, 121, 1936

[12] Gould A., Rix H.-W., 2000, ApJ, 532, L32

[13] Haehnelt M.G., Kauffmann G., 2000a, MNRAS, 318, L35

[14] Haehnelt M.G. , Kauffmann G., 2000b, in: 'Black holes in Binaries and Galactic Nuclei" ESO workshop in Honour of Prof. R. Giacconi, Garching 1999, Eds. Woudt P. et al. , p. 364

[15] Haehnelt M.G., Natarajan P., Rees M.J., 1998, MNRAS, 300, 817

[16] Holley-Bockelmann K., Mihos, C.J., Sigurdsson S., Hernquist L., Norman C., 2002, ApJ, 567, 817 (astro-ph/0111029)

[17] Kauffmann G., Charlot S., Balogh M.L., 2002, MNRAS, submitted (astro-ph/0103130)

[18] Kauffmann G., Haehnelt M.G., 2000, MNRAS, 311, 576

[19] Klypin A., Kravtsov, A.V., Bullock James S., Primack J.R., 2001, ApJ, 554, 903

[20] Madau P., Rees M.J., 2001, ApJ, 551, L27

[21] Kormendy J., Richstone D.O., 1995, ARA&A, 33, 581

[22] Magorrian J., Tremaine, S., 1999, MNRAS, 309, 447

[23] Magorrian J., Tremaine, S., Richstone, D., Bender, R., Bower, G., Dressler, A.,Faber, S.M., Gebhardt, K. et al., 1998, AJ, 115, 2285

[24] Merritt D., Ferrarese L., 2001, in: "The Central Kpc of Starbursts and AGN", ed. Knapen J.H., Beckman J.E., Shlosman I., Mahoney T.J., San Francisco: A.S.P., 2001, p. 335. (astro-ph/0107134)

[25] Milosavljević, Merritt D., 2001, ApJ, 563, 34 (astro-ph/0103350)

[26] Moore B., Governato F., Quinn T., Stadel, J., Lake, G., 1998, ApJ, 499, 5

[27] Navarro J., Frenk C., White S.D.M., 1996, ApJ, 462, 563

[28] Ostriker J.P, 2000, Phys Rev Lett, 84, 23

[29] Peters P.C., 1964, Phys. Rev. B., 136. 1224

[30] Polnarev A.G., Rees M.J, 1994, A&A, 283, 301

[31] Poon M.Y. & Merritt D. 2002, ApJ, 568, L89 (astro-ph/0111020).

[32] Rauch K.P., Tremaine S., 1996, New Astronomy, 1, 149

[33] Ravindranath S., Ho L.C., Filipenko A.V., 2002, ApJ, 566, 801 (astro-ph/0110441)

[34] Rees, M.J., 1988, Nature, 333, 523

[35] Sigurdsson S., Rees,M.J., 1997, MNRAS, 284, 318

[36] Syer D., Ulmer A., 1999, MNRAS, 306, 35

[37] Silk J., Rees , M.J., 1998, A&A, 331, 1

[38] Tremaine S., Ostriker J.P., 1999, MNRAS, 306, 662

[39] Ullio P., Zhao H., Kamionkowski M. 2001, Phy. Rev. D54, 43504 (astro-ph/0101481)

[40] Yu Q., 2002, MNRAS, 331, 935 (astro-ph/0109530)

[41] Zhao, H.S. 1996, MNRAS, 278, 488